# A pedagogical explanation for the non-renormalizability of gravity.

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We present a short and intuitive argument explaining why gravity is non-renormalizable. The argument is based on black-hole domination of the high energy spectrum of gravity and not on the standard perturbative irrelevance of the gravitational coupling. This is a pedagogical note, containing textbook material that is widely appreciated by experts and is by no means original.

## I. INTRODUCTION

In this note we present what we perceive to be an intuitive explanation of the familiar fact that gravity is not a renormalizable quantum field theory. Since this is somehow the place where gravity and quantum mechanics clash, we felt that even though this is by now textbook material, some people (particularly students and postdocs<sup>1</sup>), may benefit from its exposition in the way done in this note.

The crux of the argument, which appears e.g. in [1], is one line long and here it is: The very-high energy spectrum of any d-dimensional quantum field theory is that of a d-dimensional conformal field theory. This is not true for gravity. The rest of this note<sup>2</sup> is devoted to explaining this short claim. We felt that real understanding had to go through a crash course in Wilsonian renormalization, explaining some facts about conformal field theories and finally pointing out why as a simple consequence of black hole thermodynamics gravity can not be a renormalizable quantum field theory.

The structure of this note is the following. We start in section II by giving a crash course in Wilsonian renormalization. In section III we discuss the perturbative irrelevance of the gravitational coupling which is the basis for the standard non-renormalization argument. In section IV we discuss some relevant aspects of the density of states in conformal field theory. In section V we present the argument for the non-renormalizability

of gravity, based on black hole entropy considerations. We conclude in section VI.

#### II. RENORMALIZATION

It turns out to be a fact of nature that the low energy behavior of many systems is largely independent of the details of what goes on in higher energy scales. For example, you need not worry about Feynman diagrams when analyzing the physics of waves in water. You need not even worry about atoms. The Navier-Stokes equations, formulated in the approximation of a continuous medium, are enough. The actual physical theory describing the molecules and their interaction does creep in through determining various coefficients in the low energy theory (e.g. the viscosity coefficient). However, once this quantity has been measured one can use the Navier-Stokes equation to make predictions about the propagation of waves in water and not worry about the underlying theory<sup>3</sup>. Those coarse grained descriptions applicable at low energies are called low energy effective field-theories. Even though field theory applications in, say, condensed matter, are sometimes referred to as being not-fundamental in contrast to, say, the standard model, a more honest statement is that they are both "just" effective descriptions applicable at different energy scales<sup>4</sup>. The LHC, soon to be turned on at CERN, is designed to probe the TEV scale where the standard model effectiveness as a low energy description is expected to break down.

The modern theory of renormalization and its application to quantum field theory was pioneered by Wilson, Fisher, Kadanoff and Wegner [2]. This subject is covered

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We were first exposed to this point of view through conversations with Tom Banks. The motivation for writing this note is the author's feeling that he missed this simple, yet central theme in his own education.

<sup>&</sup>lt;sup>2</sup> After receiving feedback to the first version of this note we feel it is important to emphasize that the argument advocated here is not the perturbative argument about the irrelevance of the gravitational coupling, which can be circumvented along the lines of Weinberg's Asymptotic Safety. The argument we use builds on a sharp contradiction between the density of states in gravity, deduced via the Bekenstein-Hawking entropy formula for black holes, and the density of states in any renormalizable quantum field theory. Even though Wilsonian renormalization is crucial to build the argument (and in fact takes up most of the text), the reader is advised to read through to the last section where the black hole argument is presented.

Of course, this is only valid as long as one works within the energy regime where it is the correct effective description. If you wish to explode a dynamite stick in water, the Navier-Stokes equation is no longer a good low energy effective theory.

<sup>&</sup>lt;sup>4</sup> Incidentally, one sometime hears that perhaps gravity should not be quantized at all because it is not a "fundamental" theory but is derived from a deeper structure, E.g [4]. One reason why it seems this is not the case is that the same argument will tell you that you should not attempt to quantize the effective field theory describing the motions of atoms in a lattice, giving rise to phonons. So if you believe in the standard model quanta, then phonons and also gravitons are equally there.

in many textbooks, e.g. [3]. The treatment of the RG equations is modelled after Polchinski's paper [5].

## A. Regularization

Naively, even the simplest quantum field theories (QFT) are useless because the answer to almost any calculation is infinite. A standard example is the 1-loop correction to the mass in scalar  $\lambda \phi^3$ 

$$\Delta m^2 = \frac{\lambda^2}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{1}{(p^2 + m^2)((p+q)^2 + m^2)} =$$

$$= finite + c \cdot \int_0^\infty \frac{dp}{p} = \infty$$
(1)

for some constant c. The reason we got this meaningless answer was the insistence on integrating all the way to infinity in momentum space. This does not make sense physically because our quantum field theory description is bound to break at some point, if not sooner then definitely<sup>5</sup> at the Planck scale  $(p \sim M_{Planck})$ . One way to make sense of the theory is to introduce a high energy cut off scale  $\Lambda$  where we think the theory stops being valid and allow only for momenta smaller than the cutoff to run in the loops. But having done that, we run into trouble with quantum mechanics, because usually, the regularized theory is no longer unitary (since we arbitrarily removed part of the phase space to which there was associated a non-zero amplitude.) We therefore want to imagine a process of removing the cutoff but leaving behind "something that makes sense." A more formal way of describing the goal is the following. We are probing the system at some energy scale  $\Lambda_R$  (namely, incoming momenta in Feynman graphs obey  $p \leq \Lambda_R$ ) while keeping in our calculations a UV $^6$  cutoff  $\Lambda$  (  $\Lambda\gg\Lambda_R$ because at the end of the day we want to send  $\Lambda \to \infty$ .) If we can make all physical observables at  $\Lambda_R$  independent of  $\Lambda$  then we can safely take  $\Lambda \to \infty$ . In practice, it is convenient to parameterize the energy scale using an RG "time" parameter flowing towards lower and lower energies

$$\Lambda(t) = \Lambda(0)e^{-t} \tag{2}$$

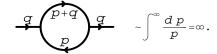


FIG. 1: The 1-loop correction to the mass in  $\phi^3$  scalar field theory is naively divergent.

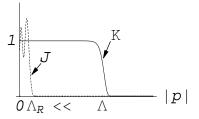


FIG. 2: The linear source term is constrained to J(p)=0 for  $p>\Lambda_R$  so as to only excite Green functions with low energy. The quadratic term contains a smooth momentum cutoff function  $\mathcal{K}(\frac{p^2}{\Lambda^2})$  with the property that  $\mathcal{K}=1$  for  $P<\Lambda$  and then falls off smoothly (and exponentially fast) to zero for  $P\geq\Lambda$ .

and demand that changing the cutoff  $\Lambda$  leaves the partition function<sup>7</sup> invariant.

$$\partial_t Z[J] = 0. (3)$$

Let us see what the consequences of such an analysis are.

# B. The Renormalization Group (RG) equations

We will model here the partition function of a quantum field theory by

$$Z[J] = I[J]/I[0] \text{ where } I[J] = \int [d\phi]e^{-\left(S_0 + S_I + S_J\right)}.$$
(4)

The action splits into the (linear) source term, the (quadratic) kinetic term and a general (polynomial) interaction term that we write in momentum space as

$$S_{J} = \int \frac{d^{4}p}{(2\pi)^{4}} J(p, \Lambda_{R}) \phi(-p)$$

$$S_{0} = \frac{1}{2} \int \frac{d^{4}p}{(2\pi)^{4}} \phi(p) \phi(-p) \frac{\Delta(p)}{\mathcal{K}(\frac{p^{2}}{\Lambda^{2}})}$$

$$S_{I} = \sum_{n=3}^{\infty} \int \frac{d^{4}p_{1}}{(2\pi)^{4}} \cdots \frac{d^{4}p_{n}}{(2\pi)^{4}} \delta^{4}(\sum p_{i})$$

$$g_{n}(p_{1}, \dots, p_{n}; \Lambda) \phi(p_{1}) \cdots \phi(p_{n}).$$
(5)

<sup>&</sup>lt;sup>5</sup> Even in free quantum field theories there is an harmonic oscillator  $\phi(x)$  in each spacetime point. Strictly within quantum field theory we can excite this degree of freedom to arbitrarily high energies. But once we add gravity to the game this can no longer be true. The reason is that when enough energy density is concentrated in a small region in space it will collapse to form a black hole.

<sup>&</sup>lt;sup>6</sup> As usual, we sometime refer to high energies (compared to some scale) as "the UV" and to low energies as "the IR". It should be kept in mind that these concepts are very context dependant. The same energy scale can be referred to as the UV in one discussion and the IR in another.

<sup>&</sup>lt;sup>7</sup> With an appropriate restriction on incoming momenta  $p < \Lambda_R$ .

where  $\Delta$  is the inverse propagator in momentum space<sup>8</sup> and we include a source function J(p) and a smooth cutoff function  $\mathcal{K}$  with specific properties that are explained
in the caption of Fig. 2.

We now demand that the partition function remains invariant upon an infinitesimal change of the high energy cutoff scale  $\Lambda$  (Eq. 3)

$$\partial_t Z[J] = \frac{1}{I^2[0]} \Big( \partial_t I[J] \cdot I[0] - I[J] \cdot \partial_t I[0] \Big) = 0. \quad (6)$$

We begin by computing the scale derivative of I[J]

$$\partial_t I[J] = \int [d\phi] \Big\{ \partial_t \left( e^{-\mathcal{S}_0 - \mathcal{S}_J} \right) e^{-\mathcal{S}_I} + e^{-\mathcal{S}_0 - \mathcal{S}_J} \partial_t \left( e^{-\mathcal{S}_I} \right) \Big\}$$
(7)

We now use the definition of functional derivatives  $\delta\phi(q)/\delta\phi(p) = \delta^4(p-q)$  to write

$$\int \frac{d^4p}{(2\pi)^4} \frac{\delta}{\delta\phi(p)} \frac{\delta}{\delta\phi(-p)} e^{-(S_0 + S_J)} =$$

$$\int \frac{d^4p}{(2\pi)^4} \frac{\delta}{\delta\phi(p)} \left( -\left[\phi \frac{\Delta}{\mathcal{K}} + J\right](p) e^{-(S_0 + S_J)} \right) =$$

$$\left[ -\delta^4(0) \cdot \int \frac{d^4p}{(2\pi)^4} \frac{\Delta}{\mathcal{K}}(p) +$$

$$\int \frac{d^4p}{(2\pi)^4} \left[\phi \frac{\Delta}{\mathcal{K}} + J\right](p) \cdot \left[\phi \frac{\Delta}{\mathcal{K}} + J\right](-p) \right] e^{-(S_0 + S_J)}$$
(8)

The first term includes an infinite factor corresponding to the volume of spacetime<sup>9</sup>, which will be cancelled by an identical but opposite contribution from the denominator. It now follows from the properties of  $J, \mathcal{K}$  explained in Fig. 2 that

- (a)  $\partial_t \mathcal{K} \cdot J = 0$  because they have disjoint support,
- (b)  $\partial_t J(p) = 0$  because J depends only on  $\Lambda_R$ ...

Using property (a) the terms that include J vanish and it follows that

$$\int \frac{d^4 p}{(2\pi)^4} \frac{\partial_t \mathcal{K}}{\Delta} \frac{\delta}{\delta \phi(p)} \frac{\delta}{\delta \phi(-p)} e^{-(\mathcal{S}_0 + \mathcal{S}_J)} = 
\left[ \int \frac{d^4 p}{(2\pi)^4} \frac{\Delta \partial_t \mathcal{K}}{\mathcal{K}^2} \phi(p) \phi(-p) - V \cdot \int \frac{d^4 p}{(2\pi)^4} \frac{\partial_t \mathcal{K}}{\mathcal{K}} \right] e^{-(\mathcal{S}_0 + \mathcal{S}_J)}.$$
(9)

Therefore, using property (b) and the fact that  $S_0$  depends on the cutoff only through the function K

$$\partial_{t}\left(e^{-(\mathcal{S}_{0}+\mathcal{S}_{J})}\right) =$$

$$-\frac{1}{2}\int \frac{d^{4}p}{(2\pi)^{4}}\phi(p)\phi(-p)\Delta \cdot \partial_{t}\left(\frac{1}{\mathcal{K}}\right)e^{-(\mathcal{S}_{0}+\mathcal{S}_{J})} =$$

$$\frac{1}{2}\int \frac{d^{4}p}{(2\pi)^{4}}\frac{\partial_{t}\mathcal{K}}{\Delta}\frac{\delta}{\delta\phi(p)}\frac{\delta}{\delta\phi(-p)}e^{-(\mathcal{S}_{0}+\mathcal{S}_{J})} + V \cdot A$$

$$(10)$$

where for brevity we call the c-number term with the explicit volume dependence  $V\cdot A$ . So we have derived that

$$\partial_t I[J] = \int [d\phi] \left\{ \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{\partial_t \mathcal{K}}{\Delta} \frac{\delta}{\delta \phi(p)} \frac{\delta}{\delta \phi(-p)} e^{-(S_0 + S_J)} + V \cdot A \right\} e^{-S_I} + e^{-S_0 - S_J} \partial_t \left( e^{-S_I} \right) \right\}$$
(11)

The c-number term  $V \cdot A$  is cancelled between the two terms in Eq. 6 because it can be pulled out of the path integral. The remaining two terms do not cancel each other but inspecting Eq. 11 we see that they can both be set to zero if we demand

$$\int [d\phi] \left\{ \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \frac{\partial_t \mathcal{K}}{\Delta} \frac{\delta}{\delta \phi(p)} \frac{\delta}{\delta \phi(-p)} e^{-(S_0 + S_J)} \right) e^{-S_I} + e^{-S_0 - S_J} \partial_t \left( e^{-S_I} \right) \right\} = 0$$
(12)

Integrating twice by parts gives

$$\partial_t (e^{-S_I}) = -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{\partial_t \mathcal{K}}{\Delta} \frac{\delta}{\delta \phi(p)} \frac{\delta}{\delta \phi(-p)} e^{-S_I} \right]. \tag{13}$$

Eq. 13 describes the infinitesimal change of the interaction Lagrangian upon changing the UV cutoff  $\Lambda$ . This dependence of the coupling constants on the cutoff is called the RG flow. The procedure of decreasing the cutoff on |p| infinitesimally from  $\Lambda$  to  $\Lambda - \delta \Lambda$  is called integrating out a "momentum shell". Notice that no infinities are encountered because all the momentum integrals are done in a finite (actually infinitesimal) range.

# C. Asymptotics of the RG flow - fixed points

Eq. 13 is a generalized version of the Heat equation

$$\partial_t \mathcal{X} = -\nabla^2 \mathcal{X} \tag{14}$$

to a space of functions with a non-trivial positive definite bilinear form  $\frac{\partial_t \mathcal{K}}{\Delta}$ , so that the Laplacian is given by  $\nabla^2 = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \frac{\partial_t \mathcal{K}}{\Delta} \frac{\delta}{\delta \phi(p)} \frac{\delta}{\delta \phi(-p)}$  and  $\mathcal{X} = e^{-S_I}$ . This kind of differential equation is called a "Gradient Flow," and is characterized by a potential function that decreases along the flow  $^{10}$ . Erroneously enough, this flow is irreversible and so the name renormalization Group is unfortunate. For the interaction Lagrangian itself Eq. 13 gives

$$\partial_t \mathcal{S}_I = (\nabla \mathcal{S}_I) \cdot (\nabla \mathcal{S}_I) - \nabla \cdot \nabla \mathcal{S}_I. \tag{15}$$

<sup>&</sup>lt;sup>8</sup> E.g. for a massive scalar field  $\Delta = p^2 + m^2$ .

<sup>&</sup>lt;sup>9</sup> Physically, this term is related to the renormalization of the vacuum energy.

 $<sup>^{10}</sup>$  The potential function here is  $V(t)=\frac{1}{2}\int[d\phi]\mathcal{X}\nabla^2\mathcal{X},$  since Eq. 14 becomes  $\dot{\mathcal{X}}=-\delta V/\delta\mathcal{X}$  and so  $\partial_t V=\frac{\delta V}{\delta\mathcal{X}}\dot{\mathcal{X}}=-(\frac{\delta V}{\delta\mathcal{X}})^2$  which is negative definite. Actually, there is a small error in this derivation because V depends on the scale also through the definition of the generalized Laplacian.

Expanding the coupling constants  $g_n(p_1, \dots, p_n; \Lambda)$  around zero momentum and labelling the whole set by some generalized index I Eq. 15 can be summarized by the  $\beta$  function equations

$$\partial_t g^I = \beta^I{}_J g^J + \beta^I{}_{JK} g^J g^K = \beta(g)$$
 (16)

which due to the form of Eq. 15 contains only linear and quadratic terms in the coupling constants  $q_I$ .

Let us pause for a quick recap. Starting from a theory defined at the cutoff  $\Lambda$  with a set of bare couplings we can get the same low energy physics at some scale  $\Lambda_R << \Lambda$  when we integrate out the momentum shell  $(\Lambda - \delta \Lambda, \Lambda)$  (i.e. lower the cutoff from  $\Lambda$  to  $\Lambda - \delta \Lambda$ ) provided that we change the coupling constants as a function of the energy scale according to Eq. 16. Keeping the scale  $\Lambda_R$  of low energy physics fixed and sending the high energy cutoff to infinity corresponds to longer and longer RG-flows. We are thus interested in the asymptotic  $t \to \infty$  behavior of Eq. 13. The heart of the argument relies on the simple fact that the asymptotic behavior of "gradient flows" is governed by fixed points. Those are the set of values for the parameters where the "time" derivative vanishes, so if we start from such a point, we stay there forever.

Since the positive function V decreases with time, as  $t \to \infty$  the limit is either V = 0, called the trivial fixed point, or some  $V = V_0 > 0$ , called a non-trivial fixed point. Non-trivial fixed points are the set of finite (or zero) values for the couplings  $g_{\star}$  that satisfy  $\beta(g_{\star}) = 0$ . The easiest example of a non-trivial fixed point is the Gaussian fixed point, or free field theory. By construction, non-trivial fixed points correspond to scale invariant field theories. As a matter of fact, most of the interesting Lorentz and scale invariant theories, end up having an even larger symmetry group, the conformal group. Such quantum field theories are called Conformal Field Theories, or CFTs for short<sup>11</sup>. The trivial fixed point corresponds to an empty theory because when couplings become infinitely large, a low energy experiment cannot excite any degree of freedom.

# D. Perturbative analysis near a fixed point

It is very instructive to analyze the RG flow in the vicinity of a fixed point.

# 1. Classical analysis

Linearizing the beta function equations (Eq. 16) around a fixed point  $g_{\star}$ , the *deviations* away from that fixed point  $D^I = g^I - g^I_{\star}$  satisfy  $\partial_t D^I = \lambda^I_{\ I} D^J$  for some

matrix  $\lambda^I_{\ J}.$  When  $\lambda^I_{\ J}$  can be diagonalized, the eigenfunctions  $\Delta^I$  satisfy  $^{12}$ 

$$\dot{\Delta}^{I} = \lambda^{I} \Delta^{I} \longrightarrow \boxed{\Delta^{I}(t) = \Delta^{I}(0) \cdot e^{\lambda^{I} t}}$$
 (17)

There are thus 3 kinds of couplings:

Relevant 
$$\lambda^{I} > 0 \Rightarrow \lim_{t \to \infty} \Delta^{I}(t) = \infty$$
  
Marginal  $\lambda^{I} = 0 \Rightarrow \Delta^{I}(t) = \Delta^{I}(0)$  (18)  
Irrelevant  $\lambda^{I} < 0 \Rightarrow \lim_{t \to \infty} \Delta^{I}(t) = 0$ .

## 2. Quantum Corrections

There are two important features missed by the classical analysis.

- Marginal operators: Quantum corrections may show that a coupling that seemed marginal classically is in fact relevant (like the gauge coupling in QCD) or is in fact irrelevant (like in scalar λφ<sup>4</sup> theory). Such couplings are called, accordingly, marginally relevant, or marginally irrelevant. A coupling that remains marginal quantum mechanically (like the gauge coupling in N = 4 Supersymmetric Yang-Mills) is called truly marginal. The existence of a truly marginal coupling signals that there is a continuous family of conformal field theories labelled by the value of a dimensionless parameter<sup>13</sup>.
- Irrelevant operators: In the classical analysis they just die away. In fact we can solve the exact non-linear equation 16 which remains quadratic in the  $\Delta$  basis

$$\partial_t \Delta^I = \lambda^I \Delta^I + \tilde{C}^I_{JK} \Delta^J \Delta^K. \tag{19}$$

The solution is

$$\Delta^I(t) = e^{\lambda^I t} \Big( \Delta^I(0) + \int_0^t ds e^{-\lambda^I s} \tilde{C}^I_{JK} \Delta^J(s) \Delta^K(s) \Big). \tag{20}$$

Given a non-zero  $\tilde{C}^I_{jk}$  with  $I \in \text{irrelevant}$  and  $j,k \in \text{relevant}$  or marginal the asymptotic value of the irrelevant parameter is completely determined by the value of the relevant and marginal couplings. It is only the initial value  $\Delta^I(0)$  of the irrelevant parameter which gets exponentially washed away.

<sup>&</sup>lt;sup>11</sup> We discuss the conformal group a little more in the last section.

<sup>&</sup>lt;sup>12</sup> This analysis is applicable, of course, only as long as  $\Delta \ll 1$ .

Even though from many perspectives there are important differences between an (ir)relevant and a marginally (ir)relevant coupling (e.g. the latter only runs logarithmically with the energy scale) as far as delivering the point aimed at in this note, we can ignore this difference. So by "(ir)relevant" we will refer to (ir)relevant or marginally (ir)relevant.

Figure 3 shows the qualitative features of such a flow in a plane spanned by one relevant and one irrelevant direction. Starting at the cutoff scale  $\Lambda$  with a set of couplings that flow so that near a fixed point there is a non-zero value to any one of the relevant couplings, we will not hit that fixed point. To make sure we hit the fixed point we need to tune our initial couplings at the cutoff so that when we flow them to the vicinity of that fixed point all the relevant couplings are exactly zero. This set of constraints defines the critical manifold for this fixed point. Tuning couplings to be on the critical manifold guarantees approaching a scale invariant behavior at low energies, which is by definition cutoff independent. This may seem trivial but in fact, is very powerful. The reason is that in practice, the vicinity of a fixed point will contain a finite and small number of relevant couplings, but an infinite number of irrelevant ones<sup>14</sup>. We can thus guarantee having a well defined limit when  $\Lambda \to \infty$  by fiddling around only with a small number of parameters.

We thus conclude that starting from any bare Lagrangian (UV) and flowing to low energies (IR) using the RG equations, we asymptotically end up either with an empty theory or a conformal field theory. This gives us a first answer to the question raised above, namely, a non trivial conformal field theory is "something that makes sense" you can be left with as an effective low energy description after removing a high energy cutoff.

# E. QFTs with a scale - dimensional transmutation

If this was the end of the story it would not be very satisfying because the theories we find in nature are not scale invariant. In order to remove the cutoff and be left with a non-scale-invariant quantum field theory we need to take a double scaling limit. What this means, is that keeping a fixed low energy scale  $\Lambda_R$  we send the high energy cutoff  $\Lambda \to \infty$ , simultaneously sending the bare relevant coupling constant closer and closer to the critical surface, in such a way that the limit gives a theory with a finite relevant coupling. To formulate this more precisely it is useful to change the definition of the RG time (Eq. 2) so that when t=0 we are at the low energy scale  $\Lambda_R$  and sending  $\Lambda \to \infty$  corresponds to starting the flow at increasingly negative times

$$\Lambda(t) = \Lambda_R e^{-t}. (21)$$

Using that, Eq. 17 gives

$$\Delta(\Lambda_R) = \left(\frac{\Lambda}{\Lambda_R}\right)^{\lambda} \cdot \Delta(\Lambda) \tag{22}$$

Therefore, we can keep a finite size<sup>15</sup> for the *relevant* (i.e.  $\lambda > 0$ ) coupling  $\Delta(\Lambda_R)$  at low energies by send-

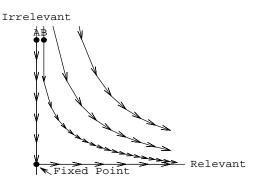


FIG. 3: The flow of coupling constants in the vicinity of a fixed point with one relevant and one irrelevant direction. Point "A" represents a theory where all the relevant couplings are exactly tuned to zero, which will therefore flow to the fixed point. Point "B" represents a theory where at least one relevant coupling is non-zero. Theory "B" will approach the fixed point for a while but then, as the relevant coupling starts to grow, it will be "kicked" out of the linear regime and will never hit the CFT.

ing  $\Lambda \to \infty$  and simultaneously sending the deviation of the bare coupling away from the fixed point  $\Delta(\Lambda) \to 0$ . More generally, starting with a cutoff  $\Lambda$  and N relevant (and marginally relevant) directions near a fixed point, we can take a double scaling limit after which we are left with a finite length scale  $\Lambda_R$  and N-1 dimensionless parameters. One relevant coupling was "traded<sup>16</sup> for a scale" giving this procedure the name dimensional transmutation. The value of the other N-1 relevant couplings encode exactly how we took the limit by fine tuning closer and closer to the critical surface. The N relevant couplings are called renormalized couplings.

Dimensional transmutation is most familiar in marginally relevant couplings. E.g. for a single marginally relevant parameter only the quadratic piece from Eq. 16 can contribute so the deviations from the fixed point obey  $\partial_t \Delta = b\Delta^2$  (instead of Eq. 17), and the solution is <sup>17</sup>

$$\frac{1}{g(\Lambda_R)} = \frac{1}{g(\Lambda)} + b \ln\left(\frac{\Lambda_R}{\Lambda}\right), \tag{23}$$

where we can again keep  $g(\Lambda_R)$  finite by sending  $g(\Lambda) \to 0$  in a correlated way to that in which we send  $\Lambda \to \infty$ .

 $<sup>^{14}</sup>$  E.g. in scalar field theory in 4 dimensions, :  $\phi^n$  : with n>4.

<sup>&</sup>lt;sup>15</sup> For Eq. 17 to be valid, the deviation  $\Delta$  must be small.

<sup>&</sup>lt;sup>16</sup> This means that we can solve for that relevant parameter in terms of the energy scale  $\Lambda_R$ , and vice versa.

We changed the notations here and denote the deviations  $\Delta$  by g so as to conform with familiar formulae where the fixed point is free field theory. Note that solving for the scales in terms of the couplings Eq. 23 gives the familiar non-perturbative expression  $\Lambda_R = \Lambda \cdot e^{-\left(\frac{1}{bg}(\Lambda) - \frac{1}{bg}(\Lambda_R)\right)}$ .

# F. Renormalizability and Universality

While conformal field theories are few and far between in the space of all possible quantum field theories, they are in this sense the "backbone" defining all other quantum field theories. A "Renormalizable" quantum field theory, describing the physics at an energy scale  $\Lambda_R$ , is a perturbation of a conformal field theory by some of its relevant operators with finite size couplings at that scale. For example, massive scalar field theory in 4 dimensions  $\mathcal{L} = \frac{1}{2} ((\partial \phi)^2 + m^2 \phi^2)$  is a perturbation by the relevant operator  $\phi^2$  of the Gaussian fixed point. Similarly, pure QCD is a perturbation of the Gaussian fixed point  $\mathcal{L} = \frac{1}{2}(\partial \mathcal{A})^2$  by the marginally relevant coupling that follows from the gauge invariant kinetic terms  $\frac{1}{4}\mathcal{F}^2$ . The values of the renormalizable couplings cannot be deduced from the bare Lagrangian because their value depends on the arbitrary way one chooses to remove the cutoff. Luckily, in many interesting cases there is just a finite and small number of relevant parameters which now encode all the physics of the theory at energy scale  $\Lambda_{R}$ , including the values of the irrelevant couplings, as was explained after Eq. 20. Having measured this finite number of couplings in the laboratory we can make unique predictions about any experiment done in the energy scale  $\Lambda_R$  and remain ignorant about the information encoded in (potentially infinitely many) irrelevant couplings. This then gives a quantitative justification to the theme advocated in the introduction, namely, the fantastic organizing principle whereby phenomenon in one energy scale are "shielded off" from the abyss of their own microscopic substructure.

In principle, it makes no difference what cutoff one uses<sup>18</sup> and how one chooses to remove it. This important principle is called *Universality*. For instance one can choose a sharp momentum cutoff by refusing to continue the integration of momenta beyond  $\Lambda$ , or a smooth cutoff as was done in this note. Other ways include replacing space by a lattice with spacing  $\Lambda^{-1}$ , or taking advantage of the analytic structure of the scattering amplitudes by using dimensional regularization. These are different "Renormalization Schemes" which are just different ways of *parameterizing* the relevant and marginal couplings, but the physics is only dictated by the fixed point around which one is perturbing.

## G. Running back to high energies

Given a renormalizable quantum field theory (point (A) in Fig. 4), we can run the RG flow backwards (i.e. towards higher energies) where the relevant coupling vanishes. Flowing further to low energies we will eventually

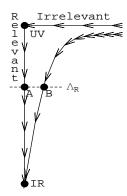


FIG. 4: Point (A) corresponds to a "renormalizable" QFT describing the physics at an energy scale  $\Lambda_R$ . A UV fixed point (e.g.  $\mathcal{L} = \frac{1}{2} \left( (\partial \phi)^2 \right)$  is perturbed by a relevant operator (e.g.  $\frac{1}{2} m^2 \phi^2$ ). Flowing further to lower energies it will eventually hit another fixed point, the IR CFT (in this e.g. it is the trivial fixed point  $m = \infty$  as can be seen in Eq. 1.). Point (B), on the other hand, corresponds to a "non-renormalizable" QFT. Adding a non-zero coupling to an irrelevant operator shoots us away from the UV fixed point we naively thought we were perturbing.

hit another fixed point, the IR conformal field theory. But what happens when we attempt to add a finite size coupling to an irrelevant operator at the scale  $\Lambda_R$  (point (B) in Fig. 4) and then attempt to run the RG flow backwards<sup>19</sup>? As can be seen in Fig. 4 we get kicked off away from the UV fixed point, the irrelevant couplings want to grow without bound. This is a signal that we are doing something wrong. Going back to the limiting process of removing the cutoff that defines a consistent unitary quantum field theory we realize that we cannot consistently remove the cutoff and retain a finite size for an irrelevant coupling at low energies. Technically we can inspect Eq. 22 with  $\lambda < 0$  and observe that keep a finite  $\Delta(\Lambda_R)$ , when  $\Lambda \to \infty$  those couplings need to grow without bound. The result of the calculation is meaningless, exactly as we now understand it should be, because we are trying to retrieve information that was lost in the irreversible gradient flow during the limiting process that is the basis for a consistent definition of any quantum field theory $^{20}$ .

<sup>&</sup>lt;sup>18</sup> In practice, calculational complexity may vary dramatically using different cutoff schemes.

<sup>&</sup>lt;sup>19</sup> E.g. when computing loop corrections in gravity.

An amusing analogy is the formal infinite answers to the illegal operation of dividing by zero. Multiplying any number x by zero is irreversible because the answer tells you nothing about x. Dividing by zero is like insisting on "undoing" it, hence the ambiguous answer. Also here the correct answer involves a more careful treatment of the limit, which sometimes does and other times does not exist.

#### H. A few remarks

Before turning our attention to a particular Lagrangian, the Einstein-Hilbert Lagrangian of General Relativity, we want to make a few remarks:

• Path integrals and the Gaussian fixed point. The derivation of the path integral formula in quantum mechanics of a massive particle involves chopping up the quantum evolution into very short time intervals and inserting complete sets of states between them. The transition amplitude between neighboring spacetime points is

$$\langle x_n, t_n | x_{n-1}, t_{n-1} \rangle \propto e^{\frac{i}{\hbar} \Delta t \left( \left( \frac{m}{2} \right) \left[ \frac{(x_n - x_{n-1})}{\Delta t} \right]^2 - V\left( \frac{(x_n + x_{n-1})}{2} \right) \right)}$$
 (24)

which is dominated by free propagation in the limit  $\Delta t \to 0$ . Because in quantum mechanics short times are associated with high energy (via the dimensionful constant  $\hbar$ ), we can rephrase the last sentence in the language of renormalization, where it reads that the UV fixed point is free. More generally, only quantum field theories that are perturbations of the Gaussian fixed point can be accurately formulated as a path integral. It is an interesting fact that all the quantum field theories useful to date in describing physical reality share this property<sup>21</sup>.

• Generality of RG equations. Even though the RG equations were derived here using a simple scalar field theory path integral, this derivation is quite general. Its relevance extends to non-trivial fixed points because to most of those one can get by starting from a Gaussian fixed point and flowing down the RG trajectory.

There are possible caveats to deriving the fixed point behavior as a rigorous theorem. One has to do with theories that include fermions, because arguments about the positivity of the inner product would fail. Another has to do with gauge theories where it may not be possible to implement a gauge invariant cutoff and define the cutoff theory using a path integral. There are various arguments suggesting that those are technical issues and the general picture advocated here still holds (see e.g. [21]).

• Dimensionality of spacetime. Renormalization theory depends in an interesting way on the dimension of spacetime. E.g, in the theory of a single scalar field in d > 6 the only relevant interaction of

the Gaussian fixed point is a mass term. In 2 dimensions there are infinitely many marginal operators because  $\phi$  is of dimension zero. In 1 spacetime dimension (i.e. quantum mechanics) there are infinitely many relevant operators. In fact any function V(x) is a relevant perturbation. This is the reason that the problems of renormalization did not appear in the old days of quantum mechanics.

- Irrelevant interactions. Here we chose to focus on a rigorous removal of the cutoff and thus the inconsistency of finite size irrelevant operators at low energies. However, in practice, irrelevant operators are very useful as a tool in a low energy effective field theory, as long as one is not pretending that they are valid all the way to infinite energies. In fact, the dimensionful couplings suppressing irrelevant operators are an indication of the scale where the low energy effective description breaks down. Marginally-irrelevant operators in particular, since they run only logarithmically with the cutoff (Eq. 23), may indicate new physics only at exponentially high energies, where the validity of the theory is in any case expected to break down<sup>22</sup>.
- Condensed matter. The language, as well as much of the intuition in the theory of renormalization in quantum field theory is borrowed from the theory of second order phase transitions in condensed matter physics.

## III. GRAVITY

# A. The gravitational coupling is irrelevant

The Einstein-Hilbert action

$$S = \frac{1}{16\pi G_N} \int d^d x \sqrt{g} \mathcal{R}[g] \tag{25}$$

governing the dynamics of classical GR was arrived at via a symmetry principle, that of general coordinate invariance. In this respect it is very similar to the gauge theories one encounters in the standard model. Using the defining property  $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$  we see that the metric is dimensionless. Therefore  $\Gamma \sim g^{-1} \partial g$  is of mass dimension 1 and the Lagrangian density  $\sqrt{g}\mathcal{R} \sim \sqrt{g}(\Gamma^2 + \partial \Gamma)$  is of mass dimension 2. Thus, the scaling dimension (with energy) of Newton's constant is

$$[G_N]_E = 2 - d$$
 (26)

<sup>&</sup>lt;sup>21</sup> Some recent discussion of possible physical signatures of a non-trivial fixed point are discussed e.g. in [6].

 $<sup>^{22}</sup>$  A good example is QED. The U(1) gauge interaction is marginally-irrelevant, but that does not really matter because the energy scale where the theory is expected to break down is much higher than, say, the electroweak symmetry breaking scale, where QED is anyways incorporated into a larger theory.

It is customary to define  $16\pi G_N=2\kappa^2=(2\pi)^{d-3}l_d^{d-2}$  where  $l_d$  is the d-dimensional *Planck length*. This looks similar to a gauge theory action  $\frac{1}{4g^2}\int d^dx \mathcal{F}^2$  where  $\kappa$  plays the role of the coupling constant. Indeed, we may choose the dynamical variable to be the metric and expand around flat space<sup>23</sup> by defining  $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu}$ , the first non-vanishing contributions are schematically given by

$$S \sim \frac{1}{2\kappa^2} \int d^dx \left[ (\partial h)^2 + (\partial h)^2 h + \cdots \right]$$
 (27)

Rescaling  $h = \kappa \tilde{h}$  in a way similar to the rescaling of the gauge field in ordinary gauge theory, we get

$$S \sim \frac{1}{2} \int d^d x \left[ (\partial \tilde{h})^2 + \kappa (\partial \tilde{h})^2 \tilde{h} + \cdots \right]$$
 (28)

which looks like a perturbation of the Gaussian fixed point  $\int d^dx (\partial \tilde{h})^2$ . Classically, this is fine, and means that gravity becomes free at large distances. But quantum mechanically we observe that for d>2 the gravitational interaction is irrelevant. The previous discussion has shown that having a finite size for an irrelevant parameter at some energy scale  $\Lambda_R$  is nonsensical quantum mechanically, unless we view the action (here Eq. 25) as a low energy field theory approximation of something else. For instance, taking too literally a path integral representation for the partition function in gravity, something like  $\int [dg] e^{-\int \sqrt{g} \mathcal{R}[g]}$ , cannot make sense because path integrals are by definition descriptions of quantum field theories that are perturbations by relevant operators of Gaussian fixed points.

Trying to solve the problem by avoiding Lagrangians all together will not help. In a Hamiltonian formulation the same problem of non-renormalizability, which technically appears in Lagrangian theories as the need for infinitely many counter terms, creeps in through the back door. There the Hamiltonian constraints, needed for a consistent quantum mechanical treatment of a theory with a gauge symmetry, do not close. Attempting to close the constraint algebra by adding the "right hand side" as an additional constraint just leads to new terms and so on<sup>24</sup>.

# B. Asymptotic Safety?

At this point in the discussion it still appears to be possible that there is another, non-trivial UV fixed point to which GR is a perturbation by a relevant operator.

This scenario was dubbed "asymptotic safety" by Weinberg [7] and is reviewed e.g. in [8] (for a recent attempt in this direction see e.g. [9]). If that was the case then General Relativity is a renormalizable quantum field theory and we were deceived by using perturbation theory in the wrong variables. In the next sections we explain why this can not be the case.

# IV. ENTROPY IN CONFORMAL FIELD THEORY

The conformal group can be defined as the group of coordinate transformations preserving the form of the metric up to a scale factor  $g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \Omega^2(x)g_{\mu\nu}(x)$ . It is an extension of the Poincare group that includes scale transformation (dilatation)  $x \to \lambda x$ . The Conformal symmetry algebra in d dimensional Lorentzian space is isomorphic to SO(2,d). Since the dilatation operator does not commute with the momentum 4-vector  $P^{\mu}$  we can no longer characterize a representation of the conformal algebra by the mass operator  $M^2$  which is a Casimir of the Poincare group. For a field that is an eigenfunction of the dilatation operator  $\phi(x) \to \lambda^{\Delta} \phi(\lambda x)$ , the eigenvalue  $^{25}$   $\Delta$  is called the "scaling dimension". Two convenient ways to label states in a conformal field theory are to decompose the SO(2,d) into two different maximal subgroups:

- 1.  $SO(1, d-1) \times SO(1, 1) \subset SO(2, d)$ . In this case the SO(1, d-1) are Lorentz transformations in d dimensions and SO(1, 1) corresponds to dilatations. Fields are labelled by their Lorentz representation and scaling dimension  $\Delta$ .
- 2.  $SO(d) \times SO(2) \subset SO(2,d)$ . This is a quantization on  $R_{time} \times S^{d-1}$  (the temporal direction is the universal cover of the SO(2) component). It is the natural choice in the operator formalism which makes a distinction between space and time. In this decomposition fields are labelled by their energy E (the eigenvalue under SO(2) translations, not to be confused with the time direction in  $R^{1,d-1}$ ), and by spin quantum numbers associated with the sphere.

The two choices are related by the conformal symmetry. Starting from  $R_{time} \times S^{d-1}$  and continuing to Euclidean signature the metric is  $ds^2 = dt_E^2 + R^2 d\Omega_{d-1}^2$  (with R being the radius of the sphere). This is conformally equivalent to the metric on the Euclidean plane  $ds^2 = dr^2 + r^2 d\Omega_{d-1}^2$  if we set  $t_E = \ln(r/R)$ . From this relation we see that the Hamiltonian  $\partial_t$  on  $R_{time} \times S^{d-1}$  maps to the dilatation operator  $r\partial_r$  on  $R^{1,d-1}$ . The relation between the quantum numbers is given by  $\Delta = RE$ .

<sup>&</sup>lt;sup>23</sup> It is immaterial which background metric we choose because any smooth manifold is flat locally, so the short distance behavior of GR is always like that of flat space.

<sup>&</sup>lt;sup>24</sup> In 2 spacetime dimensions the constraint algebra does close on the Virasoro algebra, allowing for a consistent quantization of 2d gravity. This is the starting point for string perturbation theory.

 $<sup>^{25}</sup>$  This is not to be confused with the eigenfunctions  $\Delta^I$  that appeared in the discussion following Eq. 17.

Conformal invariance substantially simplifies some aspects of finite temperature field theory. Since the zero temperature field theory has no dimensionful scales in itself the temperature sets the scale for all dimensionful quantities. Using dimensional analysis and the extensivity of the energy and the entropy in d spacetime dimensions it follows that in any conformal field theory the energy (on  $R_{time} \times S^{d-1}$ ) and entropy obey

$$S = a \cdot (RT)^{d-1}, \qquad E = b \cdot R^{d-1}T^d \tag{29}$$

where a, b are some numerical coefficients. Solving for the entropy as a function of the energy one gets

$$S \sim E^{\frac{d-1}{d}}.\tag{30}$$

In the next section we compare this with the high energy spectrum of gravity.

# V. GRAVITY IS NOT A RENORMALIZABLE QUANTUM FIELD THEORY

## 1. Zero cosmological constant

As was thoroughly explained in the first part of this note, if General Relativity was a renormalizable quantum field theory then its extreme high energy behavior should be that of a conformal field theory in the appropriate number of dimensions. However, our experience with gravity has shown that once enough energy is concentrated in a given region a black hole will form. As far as our understanding goes, the high energy spectrum of GR is dominated by black holes. More technically, it is expected that in theories of gravity, black holes will provide the dominant contribution to the large energy asymptotics of the density of states as a function of the energy.

In asymptotically flat d dimensional spacetime there is a black hole solution generalizing the Schwarzschild solution

$$ds^{2} = -f(r)dt^{2} + \frac{dr^{2}}{f(r)} + r^{2}d\Omega_{d-2}^{2},$$
  

$$f(r) = 1 - \frac{\omega_{d-2}G_{N}M}{r^{d-3}},$$
(31)

where  $\omega_n = \frac{16\pi}{n \cdot Vol(S^n)}$  and M = (E) is the (ADM) energy of the black hole state. The Horizon is thus at  $r_H^{d-3} \sim G_N M$  and the entropy as a function of the energy is given by the Bekenstein-Hawking formula

$$S = \frac{A}{4G_N} \sim \frac{r_H^{d-2}}{G_N} \sim \left(Ml_d\right)^{\frac{d-2}{d-3}} \tag{32}$$

where A is the volume of the horizon and  $l_d$  is the d-dimensional Planck length. In this case  $S \sim E^{\frac{d-2}{d-3}}$  in

contrast<sup>26</sup> to Eq. 30. It therefore follows that the large energy asymptotics of the density of states in a theory of gravity in asymptotically flat spacetime is not that of any conformal field theory, and therefore, it is not a renormalizable quantum field theory.

#### 2. Non-zero cosmological constant

Asymptotically Anti de Sitter (AdS) space (the maximally symmetric solution to Einstein's equations with a negative cosmological constant  $\Lambda=-3/R_{AdS}^2$ ) also admits black hole solutions of the form  $ds^2=-f(r)dt^2+\frac{dr^2}{f(r)}+r^2d\Omega_{d-2}^2$  but in this case the function is

$$f(r) = 1 - \frac{\omega_{d-2}G_NM}{r^{d-3}} + \frac{r^2}{R_{AdS}^2}.$$
 (33)

At distances much smaller than the AdS curvature radius  $r \ll R_{AdS}$  this is almost the Schwarzschild solution (Eq.31), but asymptotically it is very different. The horizon is at the larger solution to f(r)=0. Asymptotically  $(r\gg R_{AdS})$  the horizon sits at  $r_H\sim \left(\frac{G_NM}{|\Lambda|}\right)^{\frac{1}{d-1}}$  so that the entropy is

$$S = \frac{A}{4G_N} \sim \left(\frac{MR_{AdS}^2}{l_d}\right)^{\frac{d-2}{d-1}}.$$
 (34)

Therefore, in this case  $S \sim E^{\frac{d-2}{d-1}}$ . Again, comparing this with Eq. 30 the conclusion is that gravity in d-dimensional AdS spacetime does not have the correct density of states for a d-dimensions conformal field theory and so cannot be a renormalizable quantum field theory in that sense.

However, there is a surprise here. Eq. 34, gives the correct dependence for a conformal field theory but in one less spacetime dimension. This is a consequence of the AdS/CFT correspondence [10, 11, 12] which claims a complete equivalence, or duality, between quantum gravity in AdS space and a conformal field theory (without gravity) defined, in a precise sense that will not be explained here, on the boundary of the AdS space. AdS/CFT itself is a manifestation of a modern guiding principle in quantum gravity called "holography" [13, 14, 15].

The case of de Sitter space (the maximally symmetric solution to Einstein's equations with a *positive* cosmological constant), which seems to be the one relevant for our universe is much less well understood. For once, it is not compatible with supersymmetry which was a main tool

Note also that the exponent in Eq. 32 is bigger than one, rendering the specific heat negative which reflects the thermodynamical instability of Schwarzschild black holes. In contrast, the exponent in Eq. 30 is smaller than one, resulting in a positive specific heat.

in the advancement of our understanding in other cases. There are reasons to suspect that quantum gravity in dS space may not exist in its own right, perhaps only as a meta-stable state in another theory[16, 17, 18, 19, 20].

## VI. CONCLUSIONS

In this note we tried to give a concise and hopefully intuitive explanation to the fact that gravity is not a renormalizable quantum field theory. The basic reason is that the asymptotic density of states in gravity is dominated by black holes. This leads to a behavior qualitatively different from all quantum field theories.

A concern that was repeatedly raised after the appearance of the first version of this note is that the black hole argument is an artifact of the low energy approximation, so that "strongly coupled gravity" has no blackholes and therefore may still be asymptotically safe. We believe this counter-argument does not hold because the asymptotic safety scenario is based on the assumption that gravity is a valid low energy approximation to some putative local quantum field theory. Therefore at least in its regime of validity it should be trusted. In particular it should be trusted to describe the horizons of large black holes, since as can be seen from Eq. 31 the *more* massive

a black hole is, the *lower* is the curvature at the horizon. But the Bekenstein-Hawking formula tells us that the density of states deduced from this valid approximation has a feature that is clearly in contradiction with any quantum field theory. Therefore, to argue against the non-renormalizability of gravity is really to argue against the validity of the Bekenstein-Hawking formula, which is an uphill battle. This is so much more so when taking into account the AdS/CFT correspondence which verifies the Bekenstein-Hawking entropy counting in the case of asymptotically AdS space, and presents a clear counter example to the idea that quantum gravity is a non-trivial fixed point.

It seems that gravity is a low energy effective field theory description of something else that is not a quantum field theory.

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